Exclusionary Bundling: The Motive for Mergers

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Abstract

This paper models how exclusionary bundling motivates mergers. Firms in two unrelated markets may want to merge only to bundle, even though bundling is possible without a merger. This is because merger is necessary in order to use bundling for an exclusionary purpose. Independently of a merger, firms can always improve their profits from pure bundling. In contrast, a merger is never profitable if not combined with bundling. Moreover, it is more profitable to bundle through strategic alliance than through merger in the short run. Thus, firms choose to merge only if the merger can lead to foreclosure. Although the merger results in losses to a rival in only one of the two markets, foreclosure occurs in both markets, since the other rival firm alone cannot compete against a bundle. In this framework, all mergers are *ex ante* anti-competitive.

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1 Introduction

In 2001, the European Commission (EC) blocked the $42 billion merger between GE and Honeywell, which had been approved earlier by US antitrust authorities. The primary reason to block the merger was that it would facilitate the bundling of aircraft engines and systems to which competitors would not be able to effectively respond. According to the summary from the Competition Policy Roundtable (Hewitt, 2002),

> Mergers uniting complementary products in which at least one of the parties have considerable market power could facilitate forced tying, pure bundling or analogous strategies (e.g. full line forcing) that restrict buyer choice. While such conduct may initially increase economic welfare, it could, under certain conditions, ultimately have the opposite effect if it eliminates a sufficient number of competitors and capacity from the market (p.7).

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This decision to block the merger has been criticized, since there has been little theoretical or empirical evidence to support this position. In a document submitted for the "OECD roundtable on Portfolio Effects in Conglomerate Mergers (2001)," the US Department of Justice (DOJ) states:

We found little, if any, evidence that aerospace suppliers have been able to gain significant market share through bundling tactics in the past. With respect to technological tying, we could likewise see no way to determine, ex ante, whether physically integrating engines and avionics/nonavionics systems together would have any foreclosure effect,[.....](p.21).

In fact, many studies have found that bundling is likely to increase consumer welfare through lower prices and cost savings. When bundling takes place purely to achieve efficiency, such bundling is likely to benefit consumers. In a very general framework of heterogenous and elastic demands, Armstrong and Vickers (forthcoming) show that mixed bundling leads to welfare gains if consumers need to incur an extra "shopping cost" in order to purchase products from more than one firm and consumer preferences for product brands are correlated. Often, bundling is used as a device for price discrimination or product differentiation, in which case the welfare implications are mostly ambiguous.[1] Only a few papers have shown that bundling can directly induce an anti-competitive outcome. Whinston (1990), Choi and Stefanadis (2001), and Nalebuff (2004) analyze how effectively a monopolist can use bundling to foreclose entry. Whinston (1990) shows that a multi-product firm can leverage its monopoly power in one market to another market with greater competition by "committing" to tying its monopoly product with a product in the other market. In this way, the monopolist commits to aggressive competition, which makes rival firms in the other market unable to survive.[2] Choi and Stefanadis (2001) consider the entry-deterrence effect of commitment on bundling in the framework of dynamic R&D decisions. In their model, an incumbent deters entry by committing to tying two complementary products as it lowers the R&D investment of entrants and, thus, the probability of entry. Nalebuff (2004) shows that an incumbent with market power in two markets can effectively defend both markets against entry by bundling the two goods, since bundling makes it difficult for a rival with only one of the two products to enter the market. [3]

[1] For models on bundling as a device for price discrimination in a monopoly framework, see, for example, Adams and Yellen (1976), McCain (1987), McAfee et al. (1989), DeGraba (1994), and Armstrong (1999). For models about bundling as a device for product differentiation, see Chen (1997) and Carbajo et al (1990), for example.

[2] Carbajo et al. (1990) investigates whether a monopolist has stratetic incentives to bundle even in the cases in which bundling does not affect the entry or exit decisions of rivals. They show that a firm with monopoly power in one market bundles its product with another product that is sold in competition with rivals because bundling softens competition.

[3] See also *United States v. Microsoft,* 253 F.3d 34, 87 (D.C. Cir. 2001) and Carlton and Waldman (2002) for more discussion on the use of bundling as a method of strategic foreclosure.

Yet, when it comes to the question of blocking a merger because of the possibility that it might induce an anti-competitive bundling, it is still not clear why blocking the merger should solve this problem. If bundling causes an anti-competitive outcome, why must the merger be blocked? If it is because mergers facilitate anti-competitive bundling, it must also stand that such anti-competitive bundling is impossible without mergers. Otherwise, blocking the mergers would be futile. Firms might be able to achieve a similar anti-competitive outcome by only bundling through strategic alliances instead of mergers, for instance. For bundling alone, firms do not need to merge. AT&T is currently providing bundling services for a satellite TV component with its phone and Internet components through a strategic alliance with DirecTV in order to counter the TV component in the bundling provided by cable companies. But if bundling is possible and profitable independently of mergers, why would firms want to merge just to bundle their products?

In this paper, we show that firms merge to bundle only if the merge-and-bundle strategy leads to foreclosure of rivals. All the proposed mergers are *ex ante* anti-competitive in the sense that mergers are motivated only to facilitate exclusionary bundling. Foreclosure is impossible without a merger, even though bundling is always profitable regardless of whether a merger takes place. In fact, firms profit the most from bundling through strategic alliance rather than bundling through merger. This is because mergers entail intensive price competition. Yet, the strategic advantage of a merger is that it lowers a rival's profits from enhanced competition. Thus, firms choose to merge only if the merger reduces a rival's profits enough to force the rival to withdraw from the market. In this case, blocking the merger would never reduce welfare. As mergers are not allowed, firms may bundle through strategic alliance instead, but such bundling does not induce foreclosure.

We consider markets for two unrelated products. In each market, there are two firms in competition, and firms profit only through product differentiation. The model consists of multi-stage games in which the firms in each market first decide whether to bundle their product with the other firm's product through either a merger or a strategic alliance and then determine their prices at a later stage. Regardless of a merger, pure bundling is profitable, while mixed bundling is not. When a merged firm offers mixed bundling, mixed bundling enhances price competition between the merged firm and rivals, whereas pure bundling softens competition in one of the two markets. A merger itself is never profitable. Without bundling, the two markets remain isolated, and thus, firms gain nothing from a merger. Combined with pure bundling, however, a merger can be profitable, as the merger internalizes the competition between the partners through bundling (the Cournot Effect). The merged firm offers a discount for bundles, which results in a higher market share and increased profits for the merged firm and lower profits for a rival.

Though the merge-and-bundle strategy incurs losses only to the rival in the second market, foreclosure occurs in both markets. Another rival in market 1, in fact, enjoys a short-term profit increase due to the merger. However, as the rival in the second market exits, the other rival in the first market alone cannot compete against the merged firm's bundle. This implies that when the monopolization occurs, the merged firm can continue to defend the two markets against entry since potential entrants in each market expect it to be unprofitable to enter. This entry-deterrence effect of bundling is discussed in Nalebuff (2004).[4] The main difference between this paper and Nalebuff (2004) is that in this paper, bundling itself is never exclusionary without merger. We show that for a large range of parameters, firms do not have an incentive to merge, and thus, they offer bundling through strategic alliances. In this case, bundling does not induce foreclosure. A merger occurs only if the merged firm intends to use bundling to foreclose competition. This is why antitrust authorities need to be cautious in approving a merger, rather than bundling.

The issue of how merged firms benefit from mixed bundling has been discussed in Economides (1993) and Choi (2008). Economides (1993) models equilibrium bundling decisions between two multi-product firms. In his model, each firm produces two products, and thus, it is implicitly assumed that bilateral mergers between firms of the two different products have already occurred. Choi (2008) examines the effect of the mergers on market prices and welfare. Both studies show that unilateral bundling by a merged firm is profitable as the merger internalizes the effect of price competition within the merging firms. Choi (2008) further analyzes how R&D incentives are affected by the merger. He shows that the merged firm increases its R&D level, whereas rivals lower their R&D levels.

The present paper differs from Economides (1993) and Choi (2008) mainly in two aspects. First, in this paper, firms do not always merge merely because bundling is profitable. Second, the results in Economides (1993) and Choi (2008) together imply that in their framework, mergers can be profitable even without bundling.[5] In these two studies, the source of profit is due to the merged firm's ability to internalize price competition within the merging partners, which does not require bundling. For this reason, in their framework, bundling by the merged firm does not always raise antitrust concerns. In contrast, in this paper, bundling combined with merger is a purely exclusionary device.

The novelty of this paper is to show that some mergers are purely motivated to facilitate exclusionary bundling. This paper provides theoretical support for the need for antitrust scrutiny in approving seemingly harmless conglomerate mergers when the effect of the merger combined with bundling is considered.

In this paper, the two goods are complementary, and thus, bundling the two enhances the total value of the bundled products, which is the source of profits for

[4] Peitz (2008) confirms that the same result holds in the case of differentiated product markets.

[5] In Economides (1993), when no firm offers bundling, each merged firm earns $\pi_{N,N} = \frac{8a^2(3b-5c)}{(7b-17c)^2}$. In Choi (2008), it is shown that pre-merger profits for the merging partners are $\pi_{A1}+\pi_{B1} = \frac{4a^2(b-c)}{(3b-7c)^2}$. Thus, if $b > \frac{1}{5}c\left(\sqrt{65}+20\right) \approx 5.61c$, even bilateral mergers are profitable without bundling, i.e., $\pi_{N,N} > \pi_{A1}+\pi_{B1}$. Since $b > 3c$ in their framework, this implies that for a large range of parameters, firms can improve their profits from mergers without bundling.

bundling firms. How much bundling enhances the market power of the firms depends on whether consumer preferences in the two markets are correlated. While the main framework is built on the assumption of independent preferences, the main results hold even in the case of correlated preferences. The only difference is that bundling becomes increasingly profitable for firms as the level of correlation increases. Thus, in the case of strong correlation, unmerged firms may not experience much loss even after intensive competition with the merged firm. As a result, in the case of strong correlation, foreclosure via merger may be a little more difficult to achieve than in the case of no correlation.

We also extend the model to the case of endogenous mergers. In the range of parameter where foreclosure is possible, all firms merge with the intention to foreclose competition through bundling, but bundling does not occur in equilibrium, since it is no longer profitable if rivals merge as well. In this case, mergers are only wasteful. If foreclosure is not possible, on the other hand, no merger occurs in equilibrium. Firms bundle through strategic alliances. The equilibrium outcome is Pareto optimal for firms.

The paper proceeds as follows. Section 2 outlines the model and shows that mixed bundling never improves the profits of participating firms, regardless of merger. In section 3, we compare two different ways to bundle, namely, bundling through strategic alliances and bundling through mergers. In section 4, we characterize equilibrium bundling strategies and show when firms merge to bundle. In section 5, we consider several extensions of the model. First, we analyze the case of endogenous merger decisions. We also check the robustness of the results by considering the case of correlated preferences. Lastly, we discuss the stability of strategic alliances. Section 6 concludes.

2 The Model

We use a framework that is an extension of the standard differentiated products model.[6] Consider two unrelated markets, market 1 and market 2. In each market i, $i = 1, 2$, a continuum of consumers of mass N are uniformly distributed on a unit length interval $[0, 1]$. Consumers purchase at most one unit of each product. The characteristics of consumers in the two markets are independent, and thus, a consumer's characteristics are given by $x_{12} = (x_1, x_2) \in [0, 1] \times [0, 1]$. We assume that consumer valuations for the two products, v_1 and v_2, are high enough to guarantee that the two markets are fully covered. Moreover, the two products are complementary. That is, $v_{12} \gg v_1 + v_2$, where v_{12} is the value of consuming both products. In each market i, $i = 1, 2$, there are two firms, A_i and B_i, competing á la Hotelling. Let a_{iA} and a_{iB} be the locations of firms A_i and B_i, respectively. Without loss of generality, we assume that $a_{iA} = 0, a_{iB} = 1$. Firms are engaged in price competition.

[6] See Matutes and Regibeau (1992), for example.

Let p_{ij} be the price of firm j in market i, for $i = 1, 2$, and $j = A, B$. A consumer located at x_i in market i incurs a disutility of $t_i x_i{}^2$ ($t_i(1 - x_i)^2$) from purchasing the product from firm A_i (B_i). Without loss of generality, assume that $t_1 \leq t_2$ .Then, in market i, for a given p_{ij}, there is a consumer who is indifferent between the two firms A_i and B_i, and for the consumer,

$$p_{iA} + t_i(x_i)^2 = p_{iB} + t_i(1 - x_i)^2.$$

Thus, firms' demands are

$$D_{iA}(p_{iA}, p_{iB}) = N\left\{\frac{p_{iB} - p_{iA}}{2t_i} + \frac{1}{2}\right\} \text{ and} \tag{1}$$

$$D_{iB}(p_{iA}, p_{iB}) = N\left\{\frac{p_{iA} - p_{iB}}{2t_i} + \frac{1}{2}\right\}, \text{ for } i = 1, 2. \tag{2}$$

For simplicity, we assume that the two firms in each market i are symmetric in that they have an identical constant marginal cost of production, i.e. $c_{iA} = c_{iB} = c_i$, for $i = 1, 2$. Each firm must incur a fixed cost of f_i to produce. Then, from the first-order conditions, the profit-maximizing prices are

$$p^*_{iA} = p^*_{iB} = t_i + c_i, \text{ and} \tag{3}$$

$$\pi^*_{iA} = \pi^*_{iB} = \frac{1}{2}t_i N - f_i, \text{ for } i = 1, 2. \tag{4}$$

We assume that $\frac{1}{2}t_i N \geq f_i$.

Now, consider a conglomerate merger between A_1 and A_2. Since there is no cost synergy, the merger does not alter any market conditions as long as the merger does not affect the taste parameter t_i in the two markets.[7] This parameter t_i in market i represents how much consumers value product differentiation for the product i. Thus, it is specific to each product i. For a consumer who buys good 1 from firm A_1, the key factor is how she would feel if she were to buy the product from B_1 instead. It does not matter whether good 1 is now produced by the merged entity between A_1 and A_2 or by A_1 alone. Hence, the merger cannot alter t_i, $i = 1, 2$.[8] As such, the

[7] When a merger proposal is under review by antitrust authorities, whether the merger is expected to induce substantial efficiency gain is a crucial element in making their decision to challenge the merger. However, in reality, antitrust authorities often have incomplete information regarding the true purpose and the anticipated effects of mergers.

[8] If the transportation costs are the "actual" cost of travel, the costs depend only on the travel distance but not on the type of the product. That is, $t_1 = t_2 = t$. In this case, mergers do not affect the traveling cost as long as merged firms keep shelving their products independently as before the merger. Alternatively, if the merged firms shelve the two products together, i.e., through mixed bundling, the cost of traveling is reduced by half. However, even in this case, the cost reduction is only due to bundling, not to merger. We interpret t_i as a taste parameter rather than actual traveling costs because the former represents more general cases of conglomerate mergers.

merged firm M makes its pricing decisions in the two markets independently. The merged firm's problem is to maximize

$$\pi_M = D_{1A}(p_{1A}, p_{1B})(p_{1A} - c_1) + D_{2A}(p_{2A}, p_{2B})(p_{2A} - c_2), \tag{5}$$

for which the first-order conditions are $\frac{d\pi_M}{dp_{1A}} = \frac{d\pi_{1A}}{dp_{1A}}$, and $\frac{d\pi_M}{dp_{2A}} = \frac{d\pi_{2A}}{dp_{2A}}$. The merged firm's problem is identical to the one before the merger. Thus, the merger is unprofitable. Proposition 1 shows that the merged firm gains nothing from mixed bundling as well.

Proposition 1 *Mixed bundling is unprofitable, regardless of merger.*

Proof. See the Appendix. ■

When there is no discount for a bundle, the merged firm earns exactly the same profits as before the merger. The merged firm is always tempted to offer a positive discount for consumers who buy a bundle since a small discount for a bundle increases the merged firm's market share and the merged firm can make up any possible losses from the discount by increasing the prices of stand-alone products. However, such a mixed bundling strategy also intensifies price competition with rivals. As a result, the merged firm ends up incurring losses from mixed bundling. While mixed bundling is unprofitable, pure bundling can increase the merged firm's profits. The next section shows the profitability of pure bundling.[9]

3 Pure Bundling

In this section, we examine the effect of unilateral pure bundling on firms' profits. We first consider the case in which firms offer pure bundling through a strategic alliance instead of through a merger. They may agree to sell their products only in bundles if it is more profitable. We then examine the case in which firms merge to bundle.

3.1 Strategic alliance to bundle

Suppose firms A_1 and A_2 agree to sell their products only in bundles. Unlike a merger, A_1 and A_2 will abide by the agreement only if they *both* find that bundling is at least as profitable as before the alliance. Let p_{AA} be the price of a bundle offered by A_1 and A_2. Then, $p_{AA} = p^s_{1A} + p^s_{2A}$, where p^s_{1A} and p^s_{2A} are the optimal prices that firms independently choose under the alliance. As a result of pure bundling, the two markets are now interdependent. Consumers have only two choices, namely, purchasing the two products either from A_1 and A_2 or from B_1 and B_2. This implies

[9] Most studies in the literature find that firms favor either mixed bundling over pure bundling or pure bundling over mixed bundling. In contrast, Vaubourg (2006) presents an interesting result; in equilibrium, pure bundling offered by one firm and mixed bundling offered by its rival may coexist. This explains why some firms prefer pure bundling while others prefer mixed bundling.

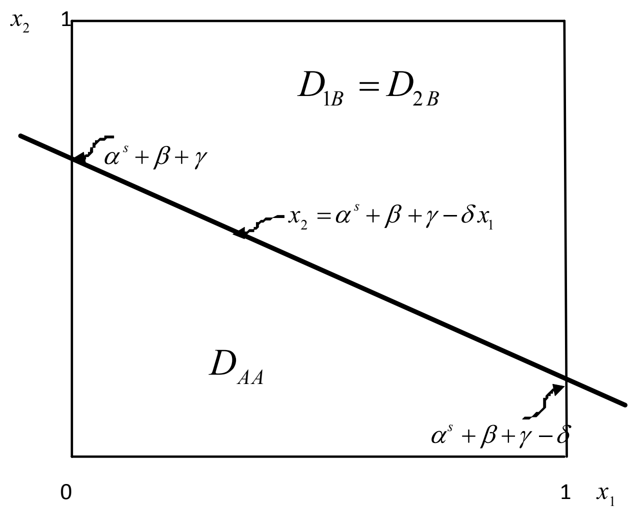

Figure 1: Demand for a Bundle: Bundling through a Strategic Alliance

that if A_1 and A_2 offer pure bundling, there is no difference whether or not B_1 and B_2 counter the bundling as well. If a consumer does not want to buy a bundle from A_1 and A_2, she must buy the two products from B_1 and B_2, regardless of whether their products are bundled. Hence, the following analysis applies to both cases, when B_1 and B_2 counter-bundle, and when they don't. Let p^s_{1B} and p^s_{2B} be the prices of products from B_1 and B_2, respectively. A consumer with x_{12} buys a bundle from A_1 and A_2 if and only if

$$\begin{aligned} & p_{AA} + t_1(x_1)^2 + t_2(x_2)^2 \leq p^s_{1B} + t_1(1-x_1)^2 + p^s_{2B} + t_2(1-x_2)^2 \\ \Leftrightarrow\quad & x_2 \leq \alpha^s + \beta + \gamma - \delta x_1, \end{aligned} \tag{6}$$

where $\alpha^s := \frac{p^s_{1B}+p^s_{2B}-p_{AA}}{2t_2}, \beta := \frac{t_1}{2t_2}, \gamma := \frac{t_2}{2t_2} = \frac{1}{2}$, $\delta := \left(\frac{t_1}{t_2}\right)$, and $\delta \leq 1$. Figure 1 depicts the demands for the bundle provided by A_1 and A_2 and the two stand-alone products, or another bundle, provided by B_1 and B_2, respectively. The demand for a bundle is

$$D_{AA}(p_{AA}, p^s_{1B}, p^s_{2B}) = D^s_{1A} = D^s_{2A} = N\left\{\alpha^s + \beta + \gamma - \frac{\delta}{2}\right\}.^{10} \tag{7}$$

The demand for good i from B_i is

$$D_{iB}(p_{AA}, p^s_{1B}, p^s_{2B}) = N\left\{1 - (\alpha^s + \beta + \gamma - \frac{\delta}{2})\right\}, \text{ for } i = 1, 2.$$

[10] If $t_1 > t_2$, $D_{AA}(p_{AA}, p^s_{1B}, p^s_{2B}) = N\{\frac{p^s_{1B}+p^s_{2B}-p_{AA}}{2t_1} + \frac{t_2}{2t_1} + \frac{1}{2} - \left(\frac{t_2}{2t_1}\right)\}$, accordingly.

The profit functions are $\pi_{iA} = D_{AA}(p_{iA} - c_i) - f_i$ and $\pi_{iB} = D_{iB}(p_{iB} - c_i) - f_i$ for $i = 1, 2$. Solving the first-order conditions, we get $\alpha^s + \beta + \gamma - \frac{\delta}{2} = \frac{1}{2}$, and thus, $D_{AA} = D_{iB} = \frac{N}{2}$. The equilibrium prices are

$$\begin{aligned} p^*_{AA} &= p^{s*}_{1A} + p^{s*}_{2A} = 2t_2 + c_1 + c_2, \\ p^{s*}_{1B} &= p^{s*}_{1A} = t_2 + c_1, \text{ and} \\ p^{s*}_{2B} &= p^{s*}_{2A} = t_2 + c_2. \end{aligned}$$

The equilibrium profits are

$$\pi^{s*}_{iA} = \pi^{s*}_{iB} = \frac{1}{2} t_2 N - f_i, \ i = 1, 2. \tag{8}$$

From (4) and (8), it is straightforward to see that $\pi^{s*}_{ij} \geq \pi^*_{ij}$, and $\sum_i \pi^{s*}_{ij} > \sum_i \pi^*_{ij}$, for all $i = 1, 2$, $j = A, B$.

Proposition 2 *Bundling through strategic alliances is always profitable if $t_1 \neq t_2$.*

As a result of pure bundling, all firms are at least weakly better off. This is because pure bundling differentiates bundled products from unbundled products or other bundled products. Pure bundling integrates the two markets. As the two products are sold only in bundles, the value of each product in a bundle is now the same as the value of the bundle. Given that consumer valuations of the two products are independent, consumers are willing to pay up to $\max\{t_1, t_2\}$ for a bundle. Hence, firms now can charge $t_2 = \max\{t_1, t_2\}$ for each component of the bundle, which generates the profits from bundling. [11]

The possibility of bundling through a strategic alliance has been discussed in Gans and King (2006) as well. In their model, firms first decide whether to agree on offering a fixed discount for consumers purchasing bundled products. Once decisions on discounts are made, firms then independently choose their prices. This sequential structure of pricing decisions, which represents the commitment of firms to reward consumer loyalty, makes it possible for firms to internalize the impact of bundled discounts, if any, on the prices of stand-alone products. As a result, bundled discount inflates the stand-alone prices, which generates the profit from bundling. But this "co-branding," i.e., bundling through a strategic alliance, is profitable only if it is unilateral. If both pairs of firms offer co-branding, no one benefits from it. In contrast, in this paper, it is always profitable to bundle through strategic alliances regardless of whether it is unilateral or bilateral. In Gans and King (2006), bundling firms raise their profits by price-discriminating against consumers who have strong preferences for only one of the two products they produce. In this paper, the profits of bundling through strategic alliances stem from product differentiation.

[11] If the valuations are correlated, consumers' willingness to pay for a bundle can increase up to $t_1 + t_2$. We discuss this case of correlated preferences in section 5.

3.2 Merging to bundle

Suppose now that A_1 and A_2 merge in order to bundle. The merged firm M offers pure bundling for the two products at the price of $\widetilde{p_M}$. Let $\widetilde{p_{1B}}$ and $\widetilde{p_{2B}}$ be the prices of rivals, B_1 and B_2, respectively. Consumers can buy both goods from M, or they can buy good 1 and good 2 from B_1 and B_2 separately. A consumer with x_{12} buys both goods from M if and only if

$$\begin{aligned} & \widetilde{p_M} + t_1(x_1)^2 + t_2(x_2)^2 \leq \widetilde{p_{1B}} + t_1(1-x_1)^2 + \widetilde{p_{2B}} + t_2(1-x_2)^2 \\ \Leftrightarrow \quad & x_2 \leq \widetilde{\alpha} + \beta + \gamma - \delta x_1, \end{aligned} \tag{9}$$

where $\widetilde{\alpha} := \frac{\widetilde{p_{1B}} + \widetilde{p_{2B}} - \widetilde{p_M}}{2t_2}$. The demand for the bundle is

$$D_M(\widetilde{p_M}, \widetilde{p_{1B}}, \widetilde{p_{2B}}) = N\left\{\widetilde{\alpha} + \beta + \gamma - \frac{\delta}{2}\right\}. \tag{10}$$

The separate demands for good i is $D_{iB}(\widetilde{p_M}, \widetilde{p_{1B}}, \widetilde{p_{2B}},) = N\left\{1 - (\widetilde{\alpha} + \beta + \gamma - \frac{\delta}{2})\right\}$, for $i = 1, 2$. Solving the first-order conditions, we obtain $\widetilde{\alpha} + \beta + \gamma - \frac{\delta}{2} = \frac{5}{8}$,and thus, $\widetilde{D_M} = \frac{5}{8}N$ and $\widetilde{D_{iB}} = \frac{3}{8}N$. The equilibrium prices are

$$\begin{aligned} \widetilde{p_M^*} &= \frac{5}{4}t_2 + c_1 + c_2 < \widetilde{p_{1B}^*} + \widetilde{p_{2B}^*}, \\ \widetilde{p_{iB}^*} &= \frac{3}{4}t_2 + c_i, \text{ for } i = 1, 2. \end{aligned}$$

The equilibrium profits are

$$\begin{aligned} \widetilde{\pi_M^*} &= \frac{25}{32}t_2 N - (f_1 + f_2), \\ \widetilde{\pi_{iB}^*} &= \frac{9}{32}t_2 N - f_i, \; i = 1, 2. \end{aligned} \tag{11}$$

Proposition 3 *If $t_1 < \frac{9}{16}t_2$, the merged firm's profit increases from pure bundling.*

While mergers allow firms to internalize the competition between merging partners, if it were not for pure bundling, the two markets would remain isolated and unrelated after merger, and thus, the merged firm would gain nothing to internalize. Combined with pure bundling, however, mergers affect pricing decisions, since pure bundling integrates the two markets into one. Bundling through merger enhances price competition. Internalizing the competition between the merging partners, M is able to offer a discount for bundles, i.e., $\widetilde{p_M^*} < \widetilde{p_{1B}^*} + \widetilde{p_{2B}^*}$. The merged firm's market share increases as a result, i.e., $\widetilde{D_M} = \frac{5}{8}N > \frac{1}{2}N$. Since the merged firm gains $\frac{1}{8}N$ of extra market share by lowering the price of two products from $t_1 + t_2$ to $\frac{5}{4}t_2$, whether or not the merge-and-bundle strategy is profitable depends on the size of the taste parameters t_2 and t_1. If t_1 is much smaller than t_2, i.e., $t_1 < \frac{9}{16}t_2$, then the merged firm's profit loss from price competition is small compared to its gain from a higher market share, and thus bundling through merger is profitable.

However, since the profitability arises from enhanced price competition, bundling through merger is not as profitable as bundling through strategic alliance. Comparing the profits from (8) and (11), we obtain

$$\widetilde{\pi_M^*} = \frac{25}{32}t_2N - (f_1 + f_2) < t_2N - (f_1 + f_2) = \pi_{1A}^{s*} + \pi_{2A}^{s*}.$$

Corollary 1 *Bundling through strategic alliance is more profitable than bundling through merger.*

When A_1 and A_2 merge to bundle, as long as the merger is profitable, i.e., $t_1 < \frac{9}{16}t_2$, B_1 benefits from the merger in the short run, since $\widetilde{p_{1B}^*}$ increases if $t_1 < \frac{3}{4}t_2$. In contrast, B_2 always incurs losses as a result of the merger, i.e., $\widetilde{\pi_{2B}^*} = \frac{9}{32}t_2N - f_2 < \pi_{2B}^* = \frac{1}{2}t_2N - f_2$. While a merger is not as profitable as a strategic alliance for A_1 and A_2 in the short run, the fact that one of the rival firms incurs losses due to the merger becomes an important motivation to merge, since the merger will be more profitable in the long run if it induces foreclosure.

Gans and King (2006) extend their analysis on bundled discounts to the case of merger as well. The main difference between this paper and Gans and King's (2006) paper emerges from the fact that in Gans and King (2006), the merged firm must commit to bundled discounts in order to improve profits. Without this commitment to discount, neither bundling nor merger is profitable. We do not require a commitment to bundle. Rather, we model firms' equilibrium bundling behavior. More importantly, we explain why firms may prefer mergers over strategic alliances when it comes to bundling. The next section discusses the incentive of firms to merge for exclusionary bundling.

4 Exclusionary Bundling under A Merger

Until now, we have assumed that A_1 and A_2 unilaterally offer bundling if they merge. However, B_1 and B_2 may also wish to form a strategic alliance to counter the bundling by the merged firm. In this section, we incorporate this possibility. We characterize equilibrium bundling strategies and analyze how bundling influences the incentive for A_1 and A_2 to merge. We will discuss the case of endogenous merger decisions in the next section.

The timing of the game between A_1 and A_2 and rivals is as follows. At stage 0, A_1 and A_2 decide whether to merge. At stage 1, firms simultaneously decide whether to bundle. At stage 2, firms simultaneously choose their prices. The following proposition summarizes the subgame perfect Nash equilibrium of this game.

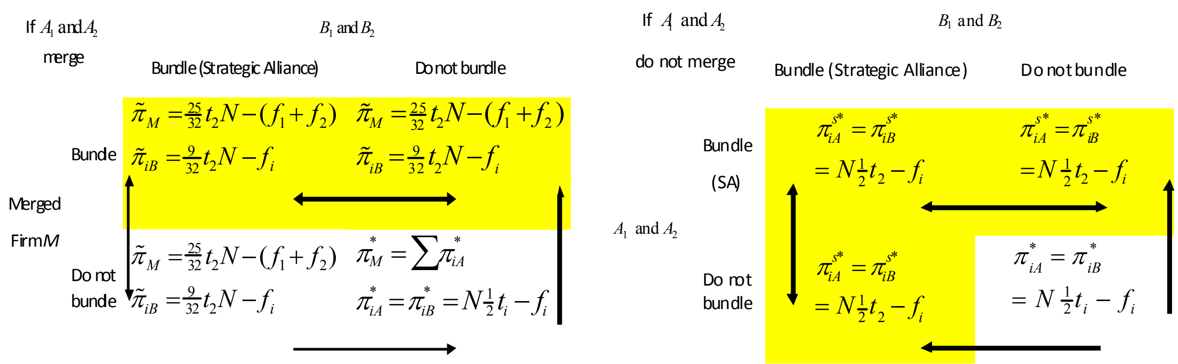

Figure 2: The payoff matrix for each subgame when foreclosure is possible

Proposition 4 *1. (Exclusionary Bundling) If $\frac{16}{9}t_1 < t_2 < f_2\frac{32}{9N}$, A_1 and A_2 merge and bundle to foreclose competition. Mergers facilitate exclusionary bundling.*

2. Otherwise, A_1 and A_2 do not merge, and at least one pair of firms offer bundling through a strategic alliance.

Proof. See the Appendix. ■

Figure 2 describes the payoff matrix for each subgame when foreclosure is possible.[12] Merger is profitable if $t_1 < \frac{9}{16}t_2$, and a rival in market 2 withdraws from the market if $\frac{9}{32}t_2N < f_2$. Thus, foreclosure is possible under merger only if $\frac{16}{9}t_1 < t_2 < f_2\frac{32}{9N}$. First, consider firms' decision to bundle in the first stage in this case. If A_1 and A_2 merge, the merged firm offers pure bundling, since pure bundling is a weakly dominant strategy for the merged firm, irrespective of rivals' strategy choices. The outcomes in this subgame always involve bundling by the merged firm. Regardless of whether rivals decide to bundle as well, the outcomes are identical: the merged firm's profit increases while B_2 incurs losses. If A_1 and A_2 do not merge, at least one pair of firms offer pure bundling through strategic alliances, but in all cases, the outcomes are identical: all the participating firms are at least weakly better off from bundling.

In equilibrium, A_1 and A_2 merge and bundle. Without merger, A_1 and A_2 would earn higher short run profits π^{s*}_{iA}, $i = 1, 2$, by bundling through a strategic alliance. Nevertheless, A_1 and A_2 choose to merge because foreclosure is infeasible without merger. Bundling provided by the merged firm significantly weakens the competitor B_2 by lowering its profits from intensive competition in market 2. As B_2 exits market 2, B_1 also becomes nonviable, since B_1 alone cannot compete with the merged firm's bundle. Consequently, the merge-and-bundle strategy induces foreclosure in both markets.

[12] For a detailed derivation of each payoff, see the proof of Proposition 4.

When foreclosure is not possible, however, merger does not occur in equilibrium. Foreclosure is not possible if either $t_1 > \frac{9}{16}t_2$, or $\frac{9}{32}t_2 N > f_2$. If $t_1 > \frac{9}{16}t_2$, the merge-and-bundle strategy is never profitable. Even if merger occurs, it is a weakly dominant strategy not to bundle. If $\frac{9}{32}t_2 N > f_2$, even if mergers are profitable, B_2 does not incur significant losses from price competition. Thus, if either $t_1 > \frac{9}{16}t_2$, or $\frac{9}{32}t_2 N > f_2$, in equilibrium, A_1 and A_2 do not merge, and both pairs of firms bundle through strategic alliances. Firms profit from bundling since the price of good 1 increases from $t_1 + c_1$ to $t_2 + c_1$. Hence, profits are at the expense of consumer surplus. Firms' profits are a pure transfer of welfare from consumer surplus. But the level of competition remains the same in both markets.

The only purpose of merger is to facilitate exclusionary bundling. Without mergers, bundling alone never induces foreclosure. This implies that, if mergers are not allowed, firms can never use bundling for an exclusionary purpose. Blocking the mergers would only deter "exclusionary" bundling. As mergers are not allowed, firms may offer bundling through strategic alliances, but such bundling will not result in foreclosure. This explains why it may be necessary to block a merger when bundling enhances the market power of the merged firm.

Another implication of this result is that if mergers are costly, the range of parameters in which firms merge to foreclose decreases. Therefore, by implementing a policy that increases merger costs, antitrust authorities could successfully lower the chances of mergers that are purely motivated to foreclose competition. Such a policy would be particularly useful if it is *ex ante* difficult to tell whether firms merge in order to achieve efficiency gains or to facilitate exclusionary bundling. If a merger is motivated by efficiency gains, an increase in merger cost would deter only relatively less efficient mergers.

5 Discussion

In this section, first we discuss the case of endogenous merger decisions. Then, we check the robustness of our main results by considering the case in which consumer preferences for the two goods are correlated. Lastly, we consider firms' incentive to deviate from a strategic alliance.

5.1 Endogenous merger

Suppose firms simultaneously decide whether to merge. Without loss of generality, we assume that the possibility of a merger is discussed between B_1 and B_2 as well as between A_1 and A_2.[13] The timing of the game between the pair of A_1 and A_2 and rivals

[13] Switching the merger partners would not make any difference in our analysis, as the two firms in each market are symmetric. That is, considering the merger between A_1 and B_2 and the merger between B_1 and A_2 will result in the same outcome as the one in the current framework.

are as follows. At stage 0, two pairs of firms, (A_1,A_2) and (B_1,B_2), simultaneously decide whether to merge. At stage 1, each pair of firms decides whether to bundle. Firms simultaneously choose their prices at stage 2.

Firms are aware that merger itself cannot be profitable. While bundling is the only way to improve their profits, bundling may not be always profitable, especially when the other pair of firms merge as well. In the Appendix, we show that if both pairs of firms merge, it is never profitable to bundle. Suppose $A_1$and A_2 merge to bundle. If B_1 and B_2 also merge, the merged entity of B_1 and B_2 earn $\pi^*_{MB} = \frac{1}{2}t_2N - (f_1 + f_2)$, which is much lower than the pre-merger profits $\sum \pi^*_{iB} = \frac{1}{2}(t_1 + t_2)N - (f_1 + f_2)$. The losses for B_1 and B_2 are much larger than what they would have had if they had not merged, i.e., $\pi^*_{MB} < \sum \widehat{\pi^*_{iB}} = \frac{18}{32}t_2N - (f_1 + f_2)$, for $i = 1, 2$. Moreover, $A_1$and A_2 also incurs the same losses, i.e., $\pi^*_{MA} = \pi^*_{MB}$. This is because a counter-merger of B_1 and B_2 only ignites a detrimental price war between the two merged firms. Both try to win the market by internalizing the effect of price competition within the merging partners, which only leads to severe price cuts and profit losses for both merged firms. The same result happens even if B_1 and B_2 do not bundle, because what triggers the price war is the counter-merger, not the counter-bundling. Since this is symmetric for $A_1$and A_2, this implies that if both pairs of firms merge, neither of the merged firms will survive in equilibrium when either of them offers bundling. If neither of them offers bundling, bilateral mergers result in the same outcome as before merger.

Figure 3 summarizes the reduced-form game of endogenous mergers when $\frac{16}{9}t_1 < t_2 < f_2\frac{32}{9N}$. Only unilateral mergers successfully induce foreclosure. When both pairs of firms merge, there are two Nash equilibria in the subgame: (bundle, bundle) and (not bundle, not bundle). Eliminating the weakly dominated strategy, we obtain (not bundle, not bundle) as the unique prediction in the subgame.

Proposition 5 *Consider a game of endogenous mergers.*

1. *If $\frac{16}{9}t_1 < t_2 < f_2\frac{32}{9N}$, the unique subgame perfect Nash equilibrium of the game is that both pairs of firms merge and never bundle.*

2. *Otherwise, the unique extensive-form trembling hand perfect Nash equilibrium (THPNE) of the game is that both pairs of firms only bundle through strategic alliances.*

Proof. See the Appendix. ■

When $\frac{16}{9}t_1 < t_2 < f_2\frac{32}{9N}$, the unique equilibrium shows the outcome of the Prisoner's Dilemma. The dominant strategy for firms is to merge, while it is most profitable for firms to only bundle through strategic alliances. If only one pair of firms merge and bundle, the other unmerging firms would be in danger of being forced to leave the market. Thus, both pairs of firms choose to merge. But if both pairs merge, bundling would only result in self-destructive price wars between the two merged

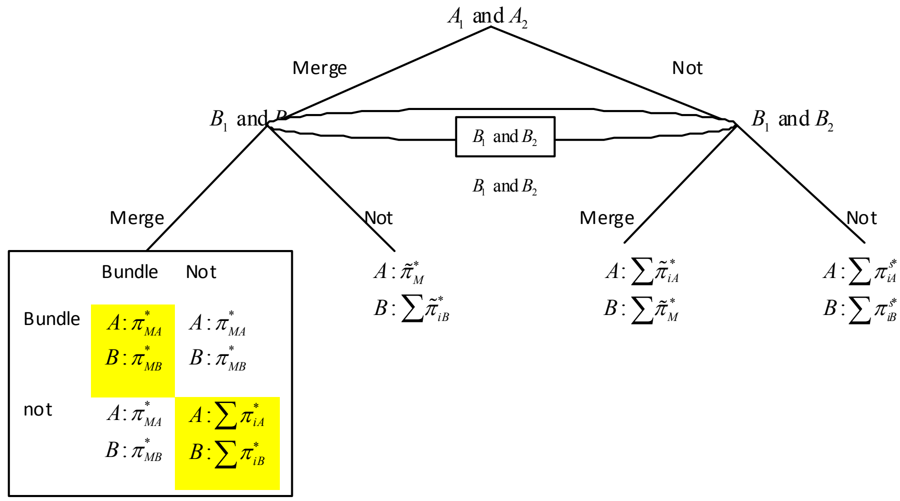

Figure 3: Reduced extensive-form game of endogenous mergers: when $\frac{16}{9}t_1 < t_2 < f_2\frac{32}{9N}$.

firms. Hence, in equilibrium, none of them decides to bundle after observing that both mergers have occurred. Since the equilibrium outcome is exactly the same as the one before the mergers, mergers are only wasteful.

However, in all other ranges of parameters, a merger is never a dominant strategy, since it is either unprofitable or sub-optimal. In equilibrium, no merger occurs, but firms offer bundling through strategic alliances. The outcome is Pareto optimal for all participating firms, since $\pi_{ij}^{s*} \geq \pi_{ij}^{*}$, for all $i = 1, 2,\ j = A, B$.[14]

5.2 Correlated preferences

For simplicity, we consider two cases: when preferences are perfectly positively correlated and when they are perfectly negatively correlated. Let $x \in [0, 1]$ be a consumer's location in market 1. Then, the characteristics of the consumer in the two markets are

$$x_{12} = \begin{cases} = (x, x), & \text{if preferences are positively correlated} \\ = (x, 1-x), & \text{if preferences are negatively correlated} \end{cases}$$

First, consider the case of positive correlation. Positive correlation implies that if a consumer is currently buying good 1 from A_1, firms can infer that she is better off

[14]Several studies report the outcome of the Prisoner's Dilemma in the framework of oligopoly bundling, implying that only unilateral bundling generates profits and equilibrium bundling is unprofitable in oligopoly. See, for example, Seidmann (1991), Economides (1993), and Denicolo (2000). In contrast, in this paper, the Prisoner's Dilemma arises only if foreclosure is possible. If not, the equilibrium under strategic alliance is Pareto optimal for firms.

buying good 2 from A_2 as well. Then, knowing this, A_1 and A_2 can now raise the price of *each* product in a bundle up to $t_1 + t_2$, which is consumers' maximum willingness to pay for the two products from A_1 and A_2. Thus, in the case of positive correlation, bundling is more profitable than it is in the case of independent valuations. On the other hand, if preferences are negatively correlated, consumers' willingness to pay for a bundle decreases to $t_2 - t_1$. However, in this case, A_1 can resume the same ability to charge up to $t_1 + t_2$ simply by pairing up with B_2 instead of A_2. Thus, irrespective of the sign of the correlation, firms can always find a way to maximize their market power for bundled products by selecting the right partner. This result is unaffected by whether bundling is offered through a merger or through a strategic alliance. The following Proposition summarizes the outcomes in the case of correlated preferences. Overall, we confirm that the main results in sections 2 through 4 remain to hold even in the case of correlated preferences.

Proposition 6 *Suppose consumer valuations of two goods are correlated.*

1. *Mergers are never profitable without bundling.*
2. *Mixed bundling is equivalent to pure bundling.*
3. *Bundling through strategic alliance is more profitable than bundling through merger.*
4. *Firms merge only if a merge-and-bundle strategy induces foreclosure, i.e., only if* $\frac{9}{7}t_1 < t_2 < f_2\frac{32}{9N} - t_1$. *Otherwise, firms offer bundling through strategic alliance.*

Proof. See the Appendix. ■

When consumer valuations of the two products are correlated, bundling unambiguously increases the equilibrium prices. If A_1 and A_2 merge and bundle, in equilibrium, the price of a bundle is $p^m = \frac{5}{4}(t_1 + t_2) + (c_1 + c_2)$, and the price of rival's stand-alone product is $p^c_{iB} = \frac{3}{4}(t_1 + t_2) + c_i$, for $i = 1, 2$. It is interesting to see that rivals' prices are higher than in the case of independent valuations. This implies that foreclosure via merger is more difficult to achieve in this case. In equilibrium, rivals earn $\pi^c_{iB} = \frac{9}{32}(t_1 + t_2)N - f_i$. B_2 incurs losses only if $t_2 > \frac{9}{7}t_1$. Even if B_2 incurs losses, A_1 and A_2 would not merge unless the losses are significant enough to induce a foreclosure. Thus, A_1 and A_2 merge only if $\frac{9}{7}t_1 < t_2 < f_2\frac{32}{9N} - t_1$, which is smaller than the range in the case of independent valuations.

5.3 Instability of strategic alliance

Since the provision of pure bundling requires that both firms agree not to sell their products individually, one may suspect instability of such an agreement between firms. Suppose A_1 and A_2 have formed a strategic alliance to bundle. We find that if B_1

and B_2 do not offer bundling, a unilateral deviation from the agreement is profitable for A_i, $i = 1, 2$.

Consider A_1's incentive to sell some of its product individually. Since selling the product at the prevailing equilibrium price $p_{1A}^{s*} = p_{1B}^{s*}$ will not generate new demand for it, the sales of individual product must be at a discount. Suppose A_1 slightly undercuts the rival's price by ε for its individual sales, i.e. $p_{1A}^{s*} = p_{1B}^{s*} - \varepsilon = t_2 - \varepsilon + c_1$. If B_1 and B_2 do not offer bundling, some consumers who otherwise would have bought the two goods from B_1 and B_2 now consider purchasing them from A_1 and B_2 for a given $\varepsilon > 0$. Along with the demand for bundles, firm A_1 gains an extra demand of $\frac{\varepsilon}{2t_1} + \frac{1}{2}$ for its individual product sales. Thus, such a deviation is always profitable. However, as it initiates price competition with B_1, in equilibrium, p_{1A}^{s*} and p_{1B}^{s*} goes down to $t_1 + c_1$, which makes it impossible to sustain the bundle price at $2t_2 + c_1 + c_2$. As the bundle price reduces to $t_1 + t_2 + c_1 + c_2$, firms cannot make profits from bundling, and thus, the strategic alliance breaks down. Therefore, a strategic alliance to bundle can be intrinsically unstable if there is a market for stand-alone products. Instability in a strategic alliance might be another reason that explains why firms prefer mergers to strategic alliances in bundling.

However, the instability of a strategic alliance exists only if B_1 and B_2 do not bundle. In sections 4 and 5.1, it has been shown that if no merger occurs, both pairs of firms bundle through strategic alliance in equilibrium. Since B_1 and B_2 bundle as well in equilibrium, even if A_1 is interested in selling its product individually, individual sales are not possible. This is because consumers cannot separately find a matching second product in the market, as the products of B_1 and B_2 are available only in bundles. Therefore, in equilibrium, strategic alliance is stable.

6 Conclusion

This paper demonstrates how seemingly harmless conglomerate mergers can become a purely exclusionary device when combined with bundling. The sole purpose of the mergers is to elicit foreclosure through bundling. In the case of a unilateral merger, even though the merger lowers market prices in the short run, it severely weakens competition in the long run by forcing rivals out in both markets. Blocking the merger only deters foreclosure because without merger, firms cannot use bundling for an exclusionary purpose. For this reason, we believe that there is a need for antitrust scrutiny in approving conglomerate mergers between complementary products producers when bundling can significantly enhance the market power of merged firms.

In this paper, we have assumed that allied firms' pricing decisions remain completely independent after they agree to offer bundling together. Under this structure, mixed bundling is not profitable since allied firms cannot coordinate their prices to increase the demand for bundled products. Thus, the bundle price is simply the sum of their stand-alone prices. However, if allied rivals can pre-commit to a bundle discount as described in Gans and King (2006), they may create an internal structure that imitates a merger. We have not considered this case because it is unclear how a merged firm will respond to rivals' bundled discount in this case. The merged firm can either pre-commit to a bundle discount as well before it decides its stand-alone prices, or wait and react to the rivals' bundled discount by setting all its prices simultaneously. Moreover, it is unclear whether allied firms will have an incentive to pre-commit to a bundle discount if the merged firm can wait and react after the level of the allied firms' bundled discount is observed. Of course, there is also an issue of increased instability of strategic alliance. Investigating how such a strategic alliance affects merger motives can be an interesting avenue for future research.

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7 Appendix

1. **Proof of Proposition 1**

Let $p_B^m$and p_{iA} for $i = 1, 2$ be the merged firm's prices for a bundle and two stand-alone products when the merged firm offers mixed bundling. Since $v_{12} \gg v_1 + v_2$, consumers always prefer buying both products to buying only one of them. A consumer with x_{12} buys a bundle from the merged firm if and only if she prefers a bundle to separate purchases of the two products. That is,

$$p_{1A} + p_{2A} - p_B^m \geq 0 \Leftrightarrow \text{Bundle} \succsim (A_1, A_2), \tag{12}$$
$$p_{1A} + p_{2B} - p_B^m + t_2 > 2t_2x_2 \Leftrightarrow \text{Bundle} \succsim (A_1, B_2), \tag{13}$$
$$p_{1B} + p_{2A} - p_B^m + t_1 > 2t_1x_1 \Leftrightarrow \text{Bundle} \succsim (B_1, A_2), \text{ and} \tag{14}$$
$$p_{1B} + p_{2B} - p_B^m + t_1 + t_2 > 2t_1x_1 + 2t_2x_2 \Leftrightarrow \text{Bundle} \succsim (B_1, B_2). \tag{15}$$

Similarly, consumers buy (A_1, B_2) if the inequality in (13) is reversed and the following conditions hold:

$$p_{1B} - p_{1A} + t_1 > 2t_1x_1 \Leftrightarrow (A_1, B_2) \succsim (B_1, B_2), \text{ and} \tag{16}$$
$$p_{1B} - p_{2B} + p_{2A} - p_{1A} + t_1 - t_2 > 2t_1x_1 - 2t_2x_2 \Leftrightarrow (A_1, B_2) \succsim (B_1, A_2) \tag{17}$$

Consumers buy (B_1, A_2) if the inequalities in (14) and (17) are reversed, and

$$p_{2B} - p_{2A} + t_2 > 2t_2x_2 \Leftrightarrow (B_1, A_2) \succsim (B_1, B_2).$$

Let x_1^0 and x_2^0 be the consumer who is indifferent between a bundle and (B_1, A_2) and the consumer indifferent between a bundle and (A_1, B_2), respectively. Then, $x_1^0 = \frac{p_{1B}+p_{2A}-p_B^m+t_1}{2t_1} = \frac{1}{2} + \frac{p_{1B}-p_{1A}+\lambda}{2t_1}$, and $x_2^0 = \frac{p_{1A}^m+p_{2B}-p_B^m+t_2}{2t_2} = \frac{1}{2} + \frac{p_{2B}-p_{2A}+\lambda}{2t_2}$, where $\lambda = p_{1A} + p_{2A} - p_B^m \geq 0$. Similarly, $x_1^1 = \frac{1}{2} + \frac{p_{1B}-p_{1A}}{2t_1}$ and $x_2^1 = \frac{1}{2} + \frac{p_{2B}-p_{2A}}{2t_2}$ are the points at which a line from equation (15) intersects a line from equation (14), and a line from equation (13), respectively. Figure 4 summarizes the market demands for a bundle and stand-alone products.

The market demands for the merged firm's bundle and stand-alone products are

$$\begin{aligned} D_B^m &= N\left\{x_1^0x_2^0 - \frac{\lambda^2}{8t_1t_2}\right\}, \\ D_{1A} &= Nx_1^1(1 - x_2^0), \text{ and} \\ D_{2A} &= Nx_2^1(1 - x_1^0), \end{aligned}$$

respectively. For simplicity, assume that $c_1 = c_2 = f_1 = f_2 = 0$ and $N = 1$ for now. Rewriting the merged firm's profit in terms of λ, we get

$$\pi_M = p_{1A}\left\{\frac{1}{2} + \Gamma_1\right\} + p_{2A}\left\{\frac{1}{2} + \Gamma_2\right\} - \lambda\Gamma_3,$$

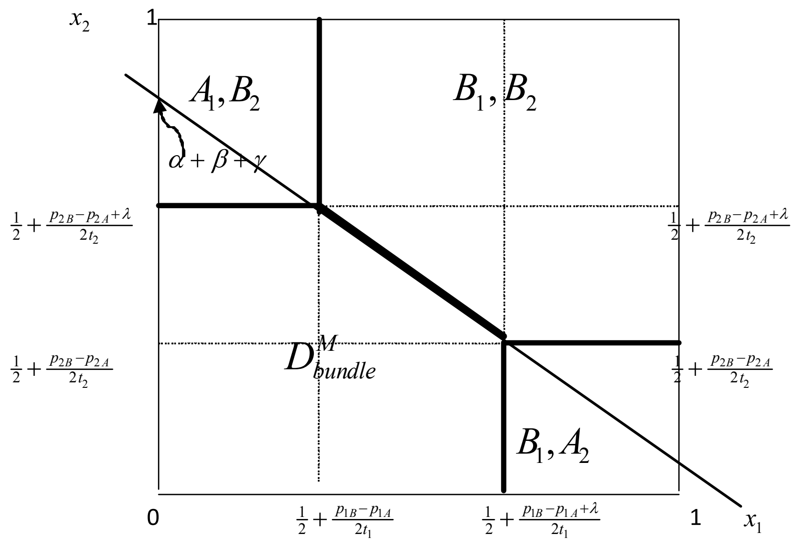

Figure 4: Market demands for a bundle and stand-alone products under mixed bundling

where $\Gamma_1 \equiv \frac{p_{1B}-p_{1A}}{2t_1}+\frac{\lambda}{2t_1}\left(\frac{1}{2}+\frac{p_{2B}-p_{2A}+\lambda}{2t_2}\right)-\frac{\lambda^2}{8t_1t_2}$, $\Gamma_2 \equiv \frac{p_{2B}-p_{2A}}{2t_2}+\frac{\lambda}{2t_2}\left(\frac{1}{2}+\frac{p_{1B}-p_{1A}+\lambda}{2t_1}\right)-\frac{\lambda^2}{8t_1t_2}$, and $\Gamma_3 \equiv \left(\frac{1}{2}+\frac{p_{1B}-p_{1A}+\lambda}{2t_1}\right)\left(\frac{1}{2}+\frac{p_{2B}-p_{2A}+\lambda}{2t_2}\right)-\frac{\lambda^2}{8t_1t_2}$. Similarly, firm B_i's profits are defined as

$$\pi_B^i = p_{iB}\left\{\frac{1}{2}-\Gamma_i\right\},\ i=1,2.$$

The proof consists of the following four steps: (1) $\lambda = 0$ is not optimal. (2) when the merged firm offers a discount for bundles (i.e., $\lambda > 0$), the optimal profits are highest when $t_1 = t_2$. As t_2 grows, the profits decrease. (3) When $t_1 = t_2$, the merged firm's profits at the optimal $\lambda^* > 0$ are lower than the profits at $\lambda = 0$. (4) When $\lambda = 0$, the merged firm's profits are the same as the sum of pre-merger profits. For all $t_1 < t_2$, merged firm incurs losses from mixed bundling.

(1) $\lambda = 0$ is not optimal.

The first order conditions are given as

$$\frac{1}{2}+\Gamma_1+\frac{\lambda}{2t_1}\left(\frac{1}{2}+\frac{p_{2B}-p_{2A}+\lambda}{2t_2}\right)-\frac{p_{1A}}{2t_1}-\frac{p_{2A}\lambda}{4t_1t_2}=0 \quad (18)$$

$$\frac{1}{2}+\Gamma_2+\frac{\lambda}{2t_2}\left(\frac{1}{2}+\frac{p_{1B}-p_{1A}+\lambda}{2t_1}\right)-\frac{p_{2A}}{2t_2}-\frac{p_{1A}\lambda}{4t_1t_2}=0 \quad (19)$$

$$\frac{(p_{1A}-\lambda)}{2t_1}\left(\frac{1}{2}+\frac{p_{2B}-p_{2A}+\lambda}{2t_2}\right)+\frac{(p_{2A}-\lambda)}{2t_2}\left(\frac{1}{2}+\frac{p_{1B}-p_{1A}+\lambda}{2t_1}\right)+\frac{\lambda^2}{4t_1t_2}-\Gamma_3=0 \tag{20}$$

$$\frac{1}{2}-\Gamma_1=\frac{p_{1B}}{2t_1} \tag{21}$$

$$\frac{1}{2}-\Gamma_2=\frac{p_{2B}}{2t_2}. \tag{22}$$

Combining the two equations (18)and (21) and the two equations (19) and (22),we get

$$2p_{1A}\lambda=\lambda^2+12t_1(t_2-p_{2B}) \tag{23}$$

$$2p_{2A}\lambda=\lambda^2+12t_2(t_1-p_{1B}), \tag{24}$$

respectively. Suppose $\lambda=0$. In this case, the optimal prices for the merged firm are the same as before merger. That is, $p_{iA}=t_i$. Evaluating the first-order condition with respect to λ at $\lambda=0$, we obtain $\frac{\partial\pi_M}{\partial\lambda}\Big|_{\lambda=0}=\frac{1}{4}>0$. Thus, $\lambda=0$ is not optimal.

(2) For $\lambda>0$, the merged firm's profits are highest when $t_1=t_2$.

Solving for an interior solution λ^* and plugging the equations (18) through (24) into π_M, we obtain

$$\begin{aligned}\pi^*_M(\lambda^*(t_1,t_2),(t_1,t_2)) &= \frac{p^{2*}_{1A}}{2t_1}+\frac{p^{2*}_{2A}}{2t_2}+\frac{2p^*_{1A}p^*_{2A}\lambda^*(t_1,t_2)}{4t_1t_2}\\ &\quad -6(t_2-p^*_{2B})\left(\frac{1}{2}+\frac{p^*_{2B}-p^*_{2A}+\lambda^*(t_1,t_2)}{2t_2}\right)\\ &\quad -6(t_1-p^*_{1B})\left(\frac{1}{2}+\frac{p^*_{1B}-p^*_{1A}+\lambda^*(t_1,t_2)}{2t_1}\right)\end{aligned}$$

where $\lambda^*(t_1,t_2)$ satisfies $\Phi(\lambda^*,t_1,t_2)=4\lambda^*\left(t_1+t_2\right)-3\lambda^{*2}-4t_1t_2$ $-\frac{64t_1^2t_2^2(t_1+t_2)\lambda^*(-11\lambda^{*4}+36t_1t_2\lambda^{*2}+216t_1^2t_2^2)+80t_1^2t_2^2\lambda^{*2}(\lambda^{*4}+76t_1t_2\lambda^{*2}-432t_1^2t_2^2)}{\left(\lambda^{*4}-76t_1t_2\lambda^{*2}+144t_1^2t_2^2\right)^2}=0$, and $p^*_{ij}\equiv p_{ij}(\lambda^*(t_1,t_2),t_1,t_2)$ is the optimal stand-alone price at $\lambda^*(t_1,t_2)>0$, for $i=1,2,j=A,B$.

Let $t_2=t_1\gamma$, where $\gamma\geq 1$. Then, the profit function $\pi^*_M(\lambda^*(t_1,t_2),(t_1,t_2))$ can be rewritten as $\pi^*_M(\lambda^*(t_1,\gamma),(t_1,\gamma))$. By Envelope Theorem,

$$\begin{aligned}\frac{\partial\pi^*_M(\lambda^*(t_1,\gamma),(t_1,\gamma))}{\partial\gamma} &= -\frac{p^{2*}_{2A}}{2t_1\gamma^2}-\frac{2p^*_{1A}p^*_{2A}\lambda^*(t_1,t_2)}{4t_1^2\gamma^2}-6t_1\underbrace{\left(\frac{1}{2}+\frac{p^*_{2B}-p^*_{2A}+\lambda^*(t_1,t_2)}{2t_1\gamma}\right)}_{B}\\ &\quad +\frac{6}{2t_1\gamma^2}\underbrace{(t_1\gamma-p^*_{2B})}_{C}\underbrace{(p^*_{2B}-p^*_{2A}+\lambda^*(t_1,t_2))}_{D}.\end{aligned} \tag{25}$$

B is x_2^0, and thus, positive since $0 \leq x_1^0$, $x_2^0 \leq 1$ by construction. C is positive. If not, when $t_2 \leq p_{2B}^*$, combined with (22), the equation (19) reduces to $\frac{3}{2t_2}(t_2 - p_{2B}^*) - \frac{p_{1A}^* \lambda^*}{4t_1 t_2} - \frac{\lambda^{*2}}{8t_1 t_2} < 0$, which implies that $t_2 \leq p_{2B}^*$ can't be true at the optimum. If D is negative, $\frac{\partial \pi_M^*(\lambda^*(t_1,\gamma),(t_1,\gamma))}{\partial \gamma} < 0$. If D is positive, combining terms in B, C, and D, we get $\frac{\partial \pi_M^*(\lambda^*(t_1,\gamma),(t_1,\gamma))}{\partial \gamma} = -\frac{p_{2A}^{2*}}{2t_1\gamma^2} - \frac{2p_{1A}^* p_{2A}^* \lambda^*(t_1,\gamma)}{4t_1^2\gamma^2} - \frac{6}{2t_1\gamma^2} p_{2B}^* \left(p_{2B}^* - p_{2A}^* + \lambda^*(t_1,\gamma)\right) - \frac{6t_1^2\gamma}{2t_1\gamma} < 0$. Therefore, $\frac{\partial \pi_M^*(\lambda^*(t_1,\gamma),(t_1,\gamma))}{\partial \gamma} < 0$ for all $t_1 > 0$ and $\gamma > 1$, and thus, $\pi_M^*(\lambda^*(t_1,t_2),(t_1,t_2))$ is highest when $\gamma = 1$, i.e., $t_1 = t_2$.

(3) When $t_1 = t_2$, $\pi_M(\lambda,(t_1,1))|_{\lambda=0} > \pi_M^*(\lambda^*(t_1,1),(t_1,1))$.

Imposing $\lambda = 0$, we get $p_{1A} = p_{1B} = t_1$ and $p_{2A} = p_{2B} = t_1$. Thus, $\pi_M(\lambda,(t_1,1))|_{\lambda=0} = \frac{1}{2}(t_1+t_1) = t_1$. On the other hand, when $t_1 = t_2$, the optimal λ^* satisfies $\Psi(\lambda^*, t_1, 1) = 576t_1^6 - 912t_1^4\lambda^{*2} - 192t_1^3\lambda^{*3} + 216t_1^2\lambda^{*4} + 52t_1\lambda^{*5} + 3\lambda^{*6} = 0$. Since $\Psi(\lambda^*, t_1, 1)|_{\lambda^*=t_1} < 0$ and $\Psi(\lambda^*, t_1, 1)|_{\lambda^*=0.8t_1} > 0$, we get $0.8t_1 < \lambda^* < t_1$. Let $\lambda^* = \theta t_1$, $0.8 < \theta < 1$. Plugging λ^* into the merged firm's profit function, we get $\pi_M^*(\lambda^*(t_1,1),(t_1,1)) = \frac{1}{5408}t_1\left(27\theta^5 + 606\theta^4 + 2112\theta^3 - 6256\theta^2 - 2928\theta + 10\,272\right)$, which is less than t_1 in the range where $0.8 < \theta < 1$. Thus, when $t_1 = t_2$, the merged firm incurs losses from offering a positive discount λ^*.

(4) Mixed bundling is never profitable.

Without bundling, the merged firm earns $\sum_i \pi_{iA}^* = \frac{1}{2}(t_1 + t_1\gamma)$. From (2), $\frac{\partial \pi_M^*(\lambda^*(t_1,\gamma),(t_1,\gamma))}{\partial \gamma} < 0$ for all t_1 and $\gamma \geq 1$, and thus, $\pi_M^*(\lambda^*(t_1,1),(t_1,1)) > \pi_M^*(\lambda^*(t_1,\gamma),(t_1,\gamma))$. Combining these results with the result from (3), we get $\sum_i \pi_{iA}^* = \frac{1}{2}(t_1 + t_1\gamma) > \pi_M(\lambda,(t_1,1))|_{\lambda=0} > \pi_M^*(\lambda^*(t_1,1),(t_1,1)) > \pi_M^*(\lambda^*(t_1,\gamma),(t_1,\gamma))$. Thus, mixed bundling is never profitable for all $\gamma \geq 1$.

If firms offer mixed bundling through a strategic alliance instead of a merger, $p_{1A} + p_{2A} = p_B^m$ by construction, and everything else is identical as before merger. Thus, mixed bundling is not profitable regardless of merger. Q.E.D.

2. **Proof of Proposition 4**

From Proposition 1, unilateral mixed bundling is never profitable regardless of whether a merger takes place. If one pair of firms offer pure bundling, the other pair of firms cannot offer mixed bundling. Thus, this is equivalent to the case in which firms only offer pure bundling. If both the merged firm (M) and allied rivals offer mixed bundling, this is equivalent to the case in which M alone offers mixed bundling since the allied rivals' bundle price is $p_{1B} + p_{2B}$ given that the allied firms' pricing decisions are independent.[15] Therefore, mixed bundling is a weakly dominated strategy for

[15] We offer an alternative way of modeling strategic alliances for future research in conclusion.

both M and allied firms. Eliminating the weakly dominated strategy, we obtain a game in which firms only choose between "pure bundling" and "no bundling."

If A_1 and A_2 merge, in the subgame four cases arise: (A) when both M and allied rivals offer bundling; (B) when M alone offers bundling; (C) when allied rivals alone offer bundling; and (D) when no firm offers bundling. We have already covered two of them in sections 2 and 3. The final outcomes of case (D) are equivalent to the pre-merger equilibrium characterized in (3) and (4), and case (B) is described in section 3.2. If A_1 and A_2 does not merge, in the sub-game there are three cases: (a) when both pairs of allies offer bundling; (b) when only one ally offers bundling; and (c) when no firm offers bundling. But as discussed in section 3.1, when at least one ally offers bundling, the outcomes are the same regardless of whether bundling is unilateral or bilateral, implying that case (a) is equivalent to case (b). Case (b) is described in section 3.1. Thus, in the following, we address cases (A) and (C) to complete the payoff structure of M and rivals at stage 2. Then, we analyze the equilibrium.

(1) When both pairs offer bundling

Suppose A_1 and A_2 merge to bundle, and B_1 and B_2 agree to bundle their products. Let p_{MB} and p_{BB} be the bundle prices of M and the allied rivals, respectively. By construction, $p_{BB} = p_{1BB} + p_{2BB}$. A consumer with $x_{12} = (x_1, x_2)$ buys a bundle from M if and only if $x_2 \leq \alpha_B + \beta + \gamma - \delta x_1$, where $\alpha_B := \frac{p_{BB} - p_{MB}}{2t_2(1-a_{2B}-a_{2A})}$. The demand for a bundle produced by M is $D_M(p_{MB}, p_{BB}) = \alpha_B + \beta + \gamma - \frac{\delta}{2}$, and the separate demand for good i is $D_{iB}(p_{MB}, p_{BB}) = 1 - (\alpha_B + \beta + \gamma - \frac{\delta}{2})$, for $i = 1, 2$.

At stage 2, solving the first-order conditions, we obtain $D_{MB} = \frac{5}{8}N$ and $D_{iBB} = \frac{3}{8}N$. Note that $D_{MB} = \widetilde{D_M}$. The equilibrium outcomes are identical to the ones in the case when M alone bundles (section 3.2). That is,

$$\begin{aligned} p^*_{MB} &= \widetilde{p^*_M} = \frac{5}{4}t_2 + c_1 + c_2, \\ &< p^*_{BB} = \sum p^*_{iBB} = \sum \widetilde{p^*_{iB}} = \frac{6}{4}t_2 + c_1 + c_2, \end{aligned}$$

$$\begin{aligned} \pi^*_{MB} &= \widetilde{\pi^*_M} = \frac{25}{32}t_2 N - (f_1 + f_2), \\ \pi^*_{iBB} &= \widetilde{\pi^*_{iB}} = \frac{9}{32}t_2 N - f_i,\ i = 1, 2. \end{aligned}$$

(2) When allied rivals alone offer pure bundling

Let $\widehat{p_{1A}}$, $\widehat{p_{2A}}$, and $\widehat{p_{BB}}$ be the prices of M's individual products in market 1 and market 2 and the ally's bundle price, respectively. By construction, $\widehat{p_{BB}} = \widehat{p_{1B}} + \widehat{p_{2B}}$, and B_1 and B_2 independently choose $\widehat{p_{1B}}$ and $\widehat{p_{2B}}$. A consumer with x_{12} buys the two products separately from M if and only if $x_2 \leq \widehat{\alpha} + \beta + \gamma - \delta x_1$, where $\widehat{\alpha} := \frac{\widehat{p_{BB}} - (\widehat{p_{1A}} + \widehat{p_{2A}})}{2t_2}$. The demand for stand-alone products produced by M is $\widehat{D_M}(\widehat{p_{1A}},\widehat{p_{2A}},\widehat{p_{BB}}) = N\left\{\widehat{\alpha}+\beta+\gamma-\frac{\delta}{2}\right\}$, and the demand for bundles is $\widehat{D_{BB}}(\widehat{p_{1A}},\widehat{p_{2A}},\widehat{p_{BB}}) = N\left\{1-(\widehat{\alpha}+\beta+\gamma-\frac{\delta}{2})\right\}$, for $i=1,2$.

At stage 2, solving the first-order conditions, we obtain $\widehat{\alpha}+\beta+\gamma-\frac{\delta}{2} = \frac{5}{8}$, $\widehat{D_M} = \frac{5}{8}N$ and $\widehat{D_{iB}} = \frac{3}{8}N$. Note that $\widehat{D_M} = \widetilde{D_M}$. The outcomes are also identical to the ones in section 3.2. That is,

$$\begin{aligned}
\widehat{p^*_{1A}}+\widehat{p^*_{2A}} &= \widetilde{p^*_M} = \frac{5}{4}t_2+c_1+c_2, \\
&< \widehat{p^*_{BB}} = \sum\widetilde{p^*_{iB}} = \frac{6}{4}t_2+c_1+c_2,
\end{aligned}$$

$$\begin{aligned}
\widehat{\pi^*_M} &= \sum\widehat{\pi^*_{iA}} = \frac{25}{32}t_2N-(f_1+f_2) = \widetilde{\pi^*_M}, \text{ and} \\
\widehat{\pi^*_{iB}} &= \widetilde{\pi^*_{iB}} = \frac{9}{32}t_2N-f_i,\ i=1,2.
\end{aligned}$$

As long as at least one pair of firms offer pure bundling, the second stage outcomes are identical regardless of who offers bundling. Hence, if A_1 and A_2 merge to bundle, the second stage payoffs are the same irrespective of whether B_1 and B_2 counters the bundling. One of the two rivals always incurs losses, i.e., $\widehat{\pi^*_{2B}} = \frac{9}{32}t_2N-f_2 < \frac{1}{2}t_2N-f_2 = \pi^*_{2B}$.

(3) Equilibrium.

Combining the results from cases (A), (C), and section 3, we can complete the payoff matrices in two subgames, namely, when A_1 and A_2 merge and when A_1 and A_2 do not merge. The payoff matrices are given in Figure 2. In each subgame, there are multiple equilibria. However, all equilibria give identical payoffs for the firms. The equilibrium follows from backward induction. A_1 and A_2 merge and monopolize the two markets. Q.E.D.

3. **Proof of Proposition 5**

In order to complete the payoff structure at stage 2, we first derive firms' payoffs when both pairs of firms merge. Then, we analyze the equilibrium.

Suppose both pairs of firms (A_1,A_2) and (B_1,B_2) merge and bundle. Let p_{MA} and p_{MB} be the market prices of the two merged firms, respectively. A consumer with x_{12} buys both goods from the merged firm of A_1 and A_2 if and only if

$$\begin{aligned}
& p_{MA}+t_1(x_1-a_{1A})^2+t_2(x_2-a_{2A})^2 \\
\leq\ & p_{MB}+t_1(x_1-(1-a_{1B}))^2+t_2(x_2-(1-a_{2B}))^2 \\
\Leftrightarrow\ & x_2 \leq \alpha_M+\beta+\gamma-\delta x_1, \qquad (26)
\end{aligned}$$

where $\alpha_M := \frac{p_{MB}-p_{MA}}{2t_2}$.

Solving the first-order conditions, we get $D_{MA} = \frac{1}{2}N$ and $D_{MB} = \frac{1}{2}N$. The equilibrium prices are

$$\begin{aligned}
p^*_{MA} &= t_2+c_1+c_2, \qquad (27) \\
p^*_{MB} &= t_2+c_1+c_2.
\end{aligned}$$

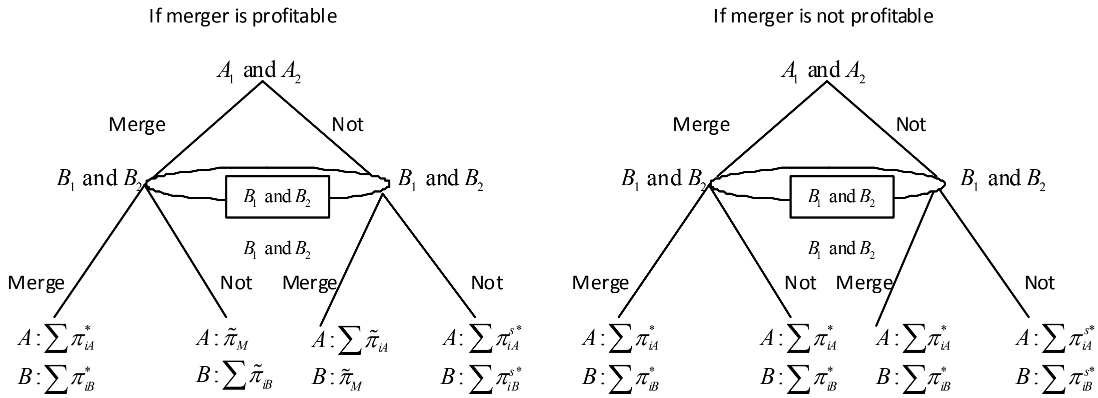

Figure 5: Reduced form extensive games when $\neg(\frac{16}{9}t_1 < t_2 < f_2\frac{32}{9N})$.

The equilibrium profits are

$$\begin{aligned}\pi^*_{MA} &= \pi^*_{MB} = \frac{1}{2}t_2N - (f_1 + f_2) \\ &< \frac{1}{2}(t_1 + t_2)N - (f_1 + f_2) = \sum \pi^*_{iA} = \sum \pi^*_{iB}. \qquad (28)\end{aligned}$$

Figure 3 shows the reduced extensive-form game when $\frac{16}{9}t_1 < t_2 < f_2\frac{32}{9N}$. When both pairs of firms merge, by eliminating the weakly dominated strategy, we get (not bundle, not bundle) as the unique prediction in the subgame. Then, the unique subgame perfect Nash equilibrium is that both pairs of firms merge, since "merge" is a dominant strategy for both. Since no merged firm bundles in equilibrium, mergers are only wasteful.

The following figure shows the reduced extensive-form games when the condition $\frac{16}{9}t_1 < t_2 < f_2\frac{32}{9N}$ is not satisfied. The parameter space is divided into two by whether $\frac{16}{9}t_1 < t_2$ or not. Suppose $\frac{16}{9}t_1 < t_2$, i.e., unilateral merger is profitable. While rival's losses are not large enough to induce foreclosure, in the subgame in which only one pair of firms merge, the merged firm always bundles, as bundling is a weakly dominant strategy for the merged firm. This case is shown in the first extensive-form game in Figure 5.

When $\frac{16}{9}t_1 < t_2$, there are two subgame perfect Nash equilibria: (Merge, Merge) and (Not, Not). Suppose $B_1$and B_2 are playing a mixed strategy of choosing to merge with probability ε for $0 < \varepsilon < 1$. Each pair of firms decides not to merge if and only if the expected profits from merging is lower than the expected joint profits from not merging. Then, for a small enough ε, i.e., $\varepsilon < \frac{\sum \pi^s_{iA} - \widetilde{\pi_M}}{\sum \pi^s_{iA} - \widetilde{\pi_M} + \sum \pi^s_{iA} - \sum \widetilde{\pi_{iA}}}$, choosing not to merge is a best response for $A_1$and A_2. By symmetry, $B_1$and B_2 also place a minimal weight on "Merge" for a small enough ε when $A_1$and A_2 are playing a mixed strategy of choosing to merge with probability ε. Thus, (Not, Not) is the unique extensive-form trembling hand perfect Nash equilibrium (THPNE).

If $\frac{16}{9}t_1 > t_2$, unilateral merger is unprofitable. In the subgame in which only one pair of firms merge, "not bundle" is a weakly dominant strategy for all firms. The new payoffs are given in the second extensive-form game in Figure 5. There are two subgame perfect Nash equilibria in this game: (Merge, Merge) and (Not, Not). Only (Not, Not) is the unique extensive-form THPNE of this game. Suppose $B_1$and B_2 are playing a mixed strategy of choosing to merge with probability ε for $0 < \varepsilon < 1$. For all $\varepsilon < 1$, choosing not to merge is a best response for $A_1$and A_2, and the same holds for $B_1$and B_2. Thus, (Not, Not) is the unique extensive-form THPNE.

Thus, when the condition $\frac{16}{9}t_1 < t_2 < f_2\frac{32}{9N}$ is not satisfied, in equilibrium, no firms merge, but they bundle through strategic alliance. Q.E.D.

4. **Proof of Proposition 6**

(1) Conglomerate mergers are never profitable without bundling.

Without bundling, the merged firm's profit function remains the same as (5). As shown in section 2, mergers are not profitable in this case.

(2) Mixed bundling is equivalent to pure bundling.

Let $p^c_{iB}$and p^c_{iA} be the prices of two stand-alone products for B_i and A_i, respectively, for $i = 1, 2$, when $A_1$and A_2 offer mixed bundling either through a strategic alliance or through a merger. Let p^m be the price of a bundle. A consumer with x buys a bundle if and only if

$$p^c_{1A} + p^c_{2A} - p^m \geq 0 \Leftrightarrow \text{Bundle} \succsim (A_1, A_2), \tag{29}$$

$$\underbrace{\frac{p^c_{1A} + p^c_{2B} - p^m}{2t_2}}_{A} + \frac{1}{2} > x \Leftrightarrow \text{Bundle} \succsim (A_1, B_2), \tag{30}$$

$$\underbrace{\frac{p^c_{1B} + p^c_{2A} - p^m}{2t_1}}_{B} + \frac{1}{2} > x \Leftrightarrow \text{Bundle} \succsim (B_1, A_2), \text{ and} \tag{31}$$

$$\underbrace{\frac{p^c_{1B} + p^c_{2B} - p^m}{2(t_1 + t_2)}}_{C} + \frac{1}{2} > x \Leftrightarrow \text{Bundle} \succsim (B_1, B_2). \tag{32}$$

Similarly, consumers buy (A_1, B_2) if

$$\underbrace{\frac{p^c_{1B} - p^c_{1A}}{2t_1}}_{D} + \frac{1}{2} > x \Leftrightarrow (A_1, B_2) \succsim (B_1, B_2), \tag{33}$$

$$x > \underbrace{\frac{p^c_{2B} - p^c_{1B} + p^c_{1A} - p^c_{2A}}{2(t_2 - t_1)}}_{E} + \frac{1}{2} \Leftrightarrow (A_1, B_2) \succsim (B_1, A_2), \tag{34}$$

and if the inequality in (30) is reversed. Consumers buy (B_1, A_2) if

$$\underbrace{\frac{p_{2B}^c - p_{2A}^c}{2t_2}}_{F} + \frac{1}{2} > x \Leftrightarrow (B_1, A_2) \succsim (B_1, B_2), \tag{35}$$

and if the inequalities in (31) and (34) are reversed.

First, we show that $C = D = F$ in equilibrium and that these conditions imply that $A = B = C$. Suppose $D > C$ and $F > C$. Then,

$$\begin{aligned} t_2(p_{1B}^c - p_{1A}^c) &> t_1(p_{1A}^c + p_{2B}^c - p^m) \text{ and} \\ t_1(p_{2B}^c - p_{2A}^c) &> t_2(p_{1B}^c + p_{2A}^c - p^m). \end{aligned}$$

Summing up these two inequalities, we find

$$(t_1 + t_2)(p_{1A}^c + p_{2A}^c - p^m) < 0,$$

which contradicts (29). Then, it must be that either $D > C > F$ or $F > C > D$. Suppose $D > C > F$. In this case, for all x in the range where $F + \frac{1}{2} < x < C + \frac{1}{2} < D + \frac{1}{2}$, bundle$\succsim(B_1, B_2) \succsim (B_1, A_2)$, and $(A_1, B_2) \succsim (B_1, B_2)$. Thus, either $(A_1, B_2) \succsim$ Bundle or Bundle $\succsim (A_1, B_2)$ in this range. If $(A_1, B_2) \succsim$ Bundle, it contradicts (29). Hence, it must be that Bundle $\succsim (A_1, B_2)$ in this range. This implies that $A > C$. When $A > C$, $2t_2(p_{1A}^c - p_{1B}^c) + 2t_1(p_{1A}^c + p_{2B}^c - p^m) > 0$, implying that $A > D$ as well. Then, given that $A > D > C > F$, for all x in the range where $C + \frac{1}{2} < x < D + \frac{1}{2} < A + \frac{1}{2}$, we get $(A_1, B_2) \succsim (B_1, B_2) \succsim$ Bundle $\succsim (A_1, B_2)$, which is a contradiction. Therefore, it cannot be that $D > C > F$. Similarly, it cannot be that $F > C > D$, either. Thus, in equilibrium, $C = D = F$.

Then, from the condition $C = D = F$, we obtain $p_{1A}^c + p_{2A}^c - p^m = 0$. Moreover, the conditions $C = D = F$ and $p_{1A}^c + p_{2A}^c - p^m = 0$ together imply that $A = B = C$, and thus, we get $A = B = C = D = E = F$. Therefore, all consumers with $x \leq C$ buy the two products from (A_1, A_2), and all other consumers with $x > C$ buy the two products from (B_1, B_2). Thus, in the case of correlated preferences, mixed bundling is equivalent to pure bundling.

(3) Bundling through strategic alliance is more profitable than bundling through merger.

The demand for a bundle provided by (A_1, A_2) is $(C+\frac{1}{2})N = \left\{\frac{p_{1B}^c + p_{2B}^c - p^m}{2(t_1+t_2)} + \frac{1}{2}\right\} N = X^c N$, and the demand for the two products provided by (B_1, B_2) is $(1 - X^c)N$. If A_1 and A_2 merge, the merged firm's profit function is $\pi_M^c = X^c N(p^m - c_1 - c_2) - (f_1 + f_2)$ and the profit function of B_i is $\pi_{Bi}^c = (1 - X^c)N(p_{iB}^c - c_i) - f_i$. From the first-order conditions, we get $X^c = \frac{5}{8}$, $p^m = \frac{5}{4}(t_1 + t_2) + (c_1 + c_2)$, $p_{iB}^c = \frac{3}{4}(t_1 + t_2) + c_i$, and

$$\pi_M^c = \frac{25}{32}(t_1 + t_2)N - (f_1 + f_2), \tag{36}$$

$$\pi_{iB}^c = \frac{9}{32}(t_1 + t_2)N - f_i. \tag{37}$$

The results are unaffected even if (B_1, B_2) counter-offer bundling through strategic alliance.

If (A_1, A_2) offer bundling through strategic alliance, $\pi_{iA}^c = X^c N(p_{iA}^c - c_i) - f_i$, $X^c = \frac{1}{2}$, and

$$\overline{\pi_{iA}^{sc}} = \overline{\pi_{iB}^{sc}} = \frac{1}{2}(t_1 + t_2)N - f_i.$$

Thus, $\sum \overline{\pi_{iA}^{sc}} > \sum \pi_{iA}^c$ and $\sum \overline{\pi_{iA}^{sc}} > \pi_M^c$, for $i = 1, 2$.

(4) Firms merge only if the merge-and-bundle strategy induces foreclosure, i.e., if $\frac{9}{7}t_1 < t_2 < f_2 \frac{32}{9N} - t_1$. Otherwise, firms offer bundling through strategic alliance.

Since $\sum \overline{\pi_{iA}^{sc}} > \pi_M^c$, firms do not have an incentive to merge if mergers do not induce foreclosure. Foreclosure occurs if unilateral merger is profitable and $\pi_{2B}^c < 0$. From (36), we can see that unilateral merger is always profitable. According to (37) and (4), B_2 incurs losses only if $t_2 > \frac{9}{7}t_1$. The losses are large enough if $\frac{9}{32}(t_1 + t_2)N - f_2 < 0$. Thus, foreclosure is possible if $\frac{9}{7}t_1 < t_2 < f_2 \frac{32}{9N} - t_1$. In this range, the merge-and-bundle strategy is a dominant strategy for both pairs of firms.

If both (A_1, A_2) and (B_1, B_2) merge and bundle, $X = \frac{1}{2}$, $p_{MA}^c = p_{MB}^c = (t_1 + t_2) + (c_1 + c_2)$, and

$$\pi_{MA}^c = \pi_{MB}^c = \frac{1}{2}(t_1 + t_2)N - (f_1 + f_2).$$

Therefore, in equilibrium, both pairs merge, but such mergers are wasteful. Q.E.D.